

# A Molecular Orbital Treatment of $^{13}\text{C}$ -H Spin-Spin Coupling Constant

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The correlation between the  $^{13}\text{C}$ -H spin-spin coupling constant,  $J_{\text{C-H}}$ , and the degree of hybridization of the carbon atom has been investigated both experimentally and theoretically by several authors.<sup>1-7</sup>

Here we will present a semi-quantitative approach to an interpretation of the values of the coupling constant,  $J_{\text{C-H}}$ , for some saturated and unsaturated compounds; the approach uses an extended Hückel treatment which has recently been proposed by Hoffmann.<sup>8</sup> In this method, a molecular orbital is expressed by a linear combination of valency atomic orbitals, that is, the 1s orbitals of a hydrogen atom, the 2s, 2px, 2py and 2pz orbitals of a carbon atom, and so on. The orbital energy is then given by solving the secular equation:

$$\det |H_{ij} - S_{ij}\epsilon| = 0 \quad (1)$$

where  $H_{ij}$  is a matrix element and  $S_{ij}$  is the overlap integral between the  $i$ th and  $j$ th atomic orbitals. These are evaluated by a procedure similar to those previously reported.<sup>8,9</sup>

When we assume that  $J_{\text{C-H}}$  is exclusively determined by the contact term, it is expressed by the equation:<sup>10</sup>

$$J_{\text{C-H}} = (1/h)(2\beta\hbar)^2 \gamma_{\text{C}} \gamma_{\text{H}} (1/\Delta E) \times [(2\pi^2/3)P^2(2s_{\text{C}}|\delta|2s_{\text{C}})(1s_{\text{H}}|\delta|1s_{\text{H}})] \quad (2)$$

where  $P = 2 \sum_i^{\text{occ}} a_{1s_{\text{H}}}^i b_{2s_{\text{C}}}^i$ , and where  $a_{1s_{\text{H}}}^i$  and  $b_{2s_{\text{C}}}^i$  are coefficients of the hydrogen 1s and carbon 2s orbitals in the  $i$ th molecular orbital respectively.  $\Delta E$  is the mean excitation energy, which is assumed to be a constant throughout

this treatment. Equation 2 is then simply written as:

$$J_{\text{C-H}} = AP^2 \quad (3)$$

It may be seen that  $J_{\text{C-H}}$  is proportional to the square of the bond order,  $P^2$ . The values of the calculated  $P^2$  and the observed  $J_{\text{C-H}}$  are cited in Table I. Since cyclobutane is non-planar and has two groups of a nonequivalent C-H bond, their mean value is indicated in the table. For the sake of simplicity, the calculation of  $J_{\text{C-H}}$  is carried out with respect to only one configuration, the chair form for cyclohexane, and the staggered form for ethane.

TABLE I. CALCULATED VALUES OF  $P^2$  AND OBSERVED  $J_{\text{C-H}}$

Compound	$P^2$	$J_{\text{C-H}}$ , c. p. s.
Group a Cycloalkanes and epoxide		
$\text{C}_2\text{H}_4\text{O}$	0.324	175
$\text{C}_3\text{H}_6$	0.270	161
$\text{C}_4\text{H}_8$	0.240	134
$\text{C}_5\text{H}_{10}$	0.209	128
$\text{C}_6\text{H}_{12}$	0.206	124
Group b Halomethanes and ethane		
$\text{CHCl}_3$	0.240	209
$\text{CH}_2\text{Cl}_2$	0.206	178
$\text{CH}_3\text{Cl}$	0.178	150
$\text{C}_2\text{H}_6$	0.176	126
$\text{CH}_4$	0.152	125
Group c Unsaturated compounds		
$\text{HC}\equiv\text{CH}$	0.377	248
$\text{C}_6\text{H}_5^{13}\text{CHO}$	0.336	174
Benzene	0.268	159
$\text{CH}_2=\text{CH}_2$	0.248	157

In each group, a, b and c, one can find a linear relationship between the values of  $P^2$  and  $J_{\text{C-H}}$ , with a few discrepancies. These discrepancies may partly be attributed to the parametrization of the matrix element in an extended Hückel method, and partly also to the fact that only the contact term is taken into account as contributing to the coupling constant.

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